Sensitivity Analysis of Anti-resonance Frequency for Vibration Test Control of a Fixture

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The test specimen in environmental vibration test is connected to the fixture through several attachment points. The forces generated by the shaker must be transmitted equally to all attachment points. The forces transmitted to attachment points, however, are different because of the flexural vibration of the fixture. The variations of the transmitted force cause the under-test, especially at anti-resonance frequencies, in vibration test control. Anti-resonance frequencies at the attachment points of the fixture must be same in order to avoid the under-test in vibration test control. The structural modification of the fixture is needed so that anti-resonance frequencies at attachment points have the same value. In this paper, the method to calculate the anti-resonance frequencies and those sensitivities is presented. This sensitivity analysis is applied to the structural modification of the fixture excited at multi-points by the shaker. The anti-resonance frequencies at the attachment points of the fixture can have the same value after structural modification, and the under-test in the vibration test control can be removed. Several computer simulations show that the proposed method can remove the under-tests, which are not removed in conventional vibration test control.

Key Words : Sensitivity Analysis, Anti-resonance Frequency, Transmitted Force, Under-test, Structural Modification, Vibration Test Fixture, Shaker

1. Introduction

In environmental vibration test, the shaker with large capacity is used to excite the whole machines. And the vibration test fixture is usually inserted between the test specimen and the shaker. The fixture is connected with specimen at several attachment points and is also connected with shaker at several points. The force generated by the shaker must be transmitted equally to all the specimen attachment points. The infinitely rigid fixture can transmit the forces equally to all specimen attachment points since all points on the fixture have same response spectra. In most practical cases, however, flexural fixture is usually used to reduce the weight of fixture. The spectra at each specimen attachment points of the flexural fixture will be quite different from the specified reference spectrum because of the flexural vibration of the fixture. The spectral difference will be magnified at resonance and anti-resonance frequencies of the fixture, which causes over-test at resonance frequencies and under-test at anti-resonance frequencies. If the magnitudes of response spectra at the attachment points are all same, the responses at those points can be equal to the reference value by controlling the forces from the shaker. At anti-resonance frequencies, some points will have zero magnitudes and the others will have non-zero magnitudes, which leads the ratios of magnitudes be infinite, while the ratios of

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magnitudes are not infinite at resonance frequencies. Therefore the worst case in the vibration control of fixture will occur at anti-resonance frequencies rather than at resonance frequencies. Controlling anti-resonance frequencies is more important than controlling resonance frequencies in the vibration control of fixture.

Recently, the averaged spectrum (Shurtleff, 1967; Bekman, 1968; Schaton, 1969; deSilva, 1993) with feedback control system is utilized in vibration test system to minimize the effects of variations of the fixture response. The optimal reference spectrum (Kim, 1996) is also studied to minimize the spectral deviation between the specified reference spectrum and spectra at the specimen attachment points. At present, however, no technique is available for removing the under-tests at anti-resonance frequencies, although the over-tests at resonance frequencies can be compensated by vibration test control system. The reason is that resonance frequencies are global characteristics and are independent of the forcing inputs and response points. However, the anti-resonance frequencies are local characteristics and depend on the particular input vector and response points. The under-tests at anti-resonance frequencies can not be compensated by vibration test control system after all. Therefore, structural dynamic modification of fixture is required in design stage of fixture to coincide all the anti-resonance frequencies at each specimen attachment points.

In case of single-input system, several methods (Shepard, 1985; Afolabi, 1987; Kajiwara, 1993; He, 1995) are studied to move the anti-resonance frequencies to a desired value. But these methods can not be applied to the multi-input system like a fixture to which excitation force is transmitted through many bolts which connect the armature table and the fixture.

In this paper, the fixture is modeled by the finite element analysis. And the method to calculate sensitivities of the anti-resonance frequencies from the inertia and stiffness matrices is presented. The structural dynamic modification of the fixture by sensitivity analysis of the anti-resonance frequencies is analytically done in order to remove the under-tests at specimen attachment points on the fixture in vibration test control. A simple aluminium plate is used as fixture in computer simulation. Length and thickness of the fixture are used as design variables. The numerical results show that the under-tests at anti-resonance frequencies can be removed in the fixture modified by the proposed method.

2. Theory

2.1 Anti-resonance frequencies

The steady state response of an n-DOF system, $\{y(t)\}=\{Y\}e^{jwt}$, subject to $\{f(t)\}=\{F\}e^{jwt}$ is expressed as the solution of following equation.

$$([K]-w^2[M])\{Y\}=\{F\} \tag{1}$$

or, in the dynamic stiffness format as

$$[Z(w)]\{Y\}=\{F\} \tag{2}$$

where $[M]$ are $[K]$ the $(n\times n)$ inertia and stiffness matrices, and $\{F\}$ are $\{Y\}$ the magnitude of the n-dimensional applied excitation force vector and displacement response vector, respectively. The response at point i, Y_i, can be determined from the Cramer's rule as follows:

$$Y_i=\frac{det([Z(i:F)])}{det([z(w)])},\ i=1,\cdots,n \tag{3}$$

where, $[z(i;F)]$ is the matrix obtained from $[Z(w)]$ by replacing column i by $\{F\}$. The resonance frequencies obtained from $det([Z(w)])=0$ are invariable regardless of the input forces and response points. The resonance frequencies can be obtained easily from the eigenvalue problem analysis. It is necessary to set $det([Z(i;F)])=0$ to determine the anti-resonance frequencies associated with Y_i. But anti-resonance frequencies depend on the position of input forces and response points.

Anti-resonance frequencies at point i can be written as

$$\begin{aligned} &det([Z(i;F)]) \\ &=det([K(i;F)-\Omega^2[M(i;0)]])=0 \end{aligned} \tag{4}$$

and Eq. (4) may be written in standard eigenproblem form as

$$[K(i\,;\,F)]\{X\}=\Omega^2[M(i\,;\,0)]\{X\} \quad (5)$$

The anti-resonance frequency Ω can be obtained as the solutions of Eq. (5). Here, $[K(i\,;\,F)]$ and $[M(i\,;\,0)]$ are as follows:

$$[K(i\,;\,F)]=\begin{bmatrix} K_{11} & K_{12} & \cdots & K_{1i-1} & F_1 & K_{1i+1} & \cdots & K_{1n} \\ K_{21} & K_{22} & \cdots & K_{2i-1} & F_2 & K_{2i+1} & \cdots & K_{2n} \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ K_{i1} & K_{i2} & \cdots & K_{ii-1} & F_i & K_{ii+1} & \cdots & K_{in} \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ K_{n1} & K_{n2} & \cdots & K_{ni-1} & F_n & K_{ni+1} & \cdots & K_{nn} \end{bmatrix}$$

$$[M(i\,;\,0)]=\begin{bmatrix} M_{11} & M_{12} & \cdots & M_{1i-1} & 0_1 & M_{1i+1} & \cdots & M_{1n} \\ M_{21} & M_{22} & \cdots & M_{2i-1} & 0 & M_{2i+1} & \cdots & M_{2n} \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ M_{i1} & M_{i2} & \cdots & M_{ii-1} & 0 & M_{ii+1} & \cdots & M_{in} \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ M_{n1} & M_{n2} & \cdots & M_{ni-1} & 0 & M_{ni+1} & \cdots & M_{nn} \end{bmatrix}$$

2.2 Sensitivities of anti-resonance frequencies

In order to remove the under-test, the fixture must be modified so that the anti-resonance frequencies of the specimen attachment points have the same value. Sensitivity analysis of anti-resonance frequencies is necessary to modify the fixture. Let n_s, n_{ar}, and n_d be the number of the specimen attachment points, anti-resonance frequencies, and design variables, respectively. The sensitivity of the j-th anti-resonance frequency at the i-th specimen attachment point, $\Omega_{ij}(i=1, \cdots, n_s\,;\,j=1, \cdots, n_{ar})$, which is caused by the k-th design variable $x_k(k=1, \cdots, n_d)$ is given by (Nagamatsu, 1986):

$$\frac{\partial\Omega_{ij}}{\partial x_k}=\{\phi_j\}^T\left(\frac{\partial[K(i\,;\,F)]}{\partial x_k} - \Omega_{ij}^2\frac{\partial[M(i\,;\,0)]}{\partial x_k}\right)\{\phi_j\}/(2\Omega_{ij}) \quad (6)$$

where, $\{\phi_j\}$ is the j-th eigenvector of Eq. (5), while $\{\phi_j\}$ is normalized with respect to $[M]$, i.e., $\{\phi_j\}^T[M]\{\phi_j\}=1$. Let $\{\Omega_j^*\}$ denotes the j-th target anti-resonance frequency which all the anti-resonance frequencies at specimen attachment points $(i=1, \cdots, n_s)$ must be coincident. And let $[S]$ denotes the sensitivity matrix which consists of the sensitivities of anti-resonance frequencies, and $\{\Delta x_k\}$ denotes the design variables to be changed. The sensitivity equation can be approximately written by taking the first order form of the Taylor series expansion as

$$\{\Omega_{ij}\}+[S]\{\Delta x_k\}\simeq\{\Omega_j^*\} \quad (7)$$

where, $\{\Omega_{ij}\}$ is defined by an $\{(n_s\times n_{ar})\times 1\}$ column vector and $[S]$ by an $[(n_s\times n_{ar})\times n_d]$ matrix as follows:

$$[S]=\begin{bmatrix} \begin{bmatrix} \frac{\partial\Omega_{11}}{\partial x_1} & \frac{\partial\Omega_{11}}{\partial x_2} & \cdots & \frac{\partial\Omega_{11}}{\partial x_{n_d}} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial\Omega_{n_s1}}{\partial x_1} & \frac{\partial\Omega_{n_s1}}{\partial x_2} & \cdots & \frac{\partial\Omega_{n_s1}}{\partial x_{n_d}} \end{bmatrix} \\ \vdots \quad \vdots \quad \cdots \quad \vdots \\ \begin{bmatrix} \frac{\partial\Omega_{1n_{ar}}}{\partial x_1} & \frac{\partial\Omega_{1n_{ar}}}{\partial x_2} & \cdots & \frac{\partial\Omega_{1n_{ar}}}{\partial x_{n_d}} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial\Omega_{n_sn_{ar}}}{\partial x_1} & \frac{\partial\Omega_{n_sn_{ar}}}{\partial x_2} & \cdots & \frac{\partial\Omega_{n_sn_{ar}}}{\partial x_{n_d}} \end{bmatrix} \end{bmatrix}$$

If the rank of the sensitivity matrix, $[S]$, is less than the number of the design variables, n_d, $\{\Delta x_k\}$ can be determined by the pseudo inverse method, which satisfies the constraint that the square norm of the modified quantities is the minimum as follows:

$$\{\Delta x_k\}=[S]^T([S][S]^T)^{-1}(\{\Omega_j^*\}-\{\Omega_{ij}\}) \quad (8)$$

Now the modified design variables $\{x_k\}_{new}$ can be obtained from Eq. (9), and calculation has to be repeated until $\{\Omega_{ij}\}$ is converged to $\{\Omega_j^*\}$.

$$\{x_k\}_{new}=\{x_k\}_{old}+a_k\cdot\{\Delta x_k\} \quad (9)$$

where, a_k is a step size parameter which should be determined in one dimensional optimization technique. Target anti-resonance frequency Ω_j^* is set as

$$\Omega_j^*=\frac{1}{n_s}\sum_{i=1}^{n_s}\Omega_{ij},\ j=1, \cdots, n_{ar} \quad (10)$$

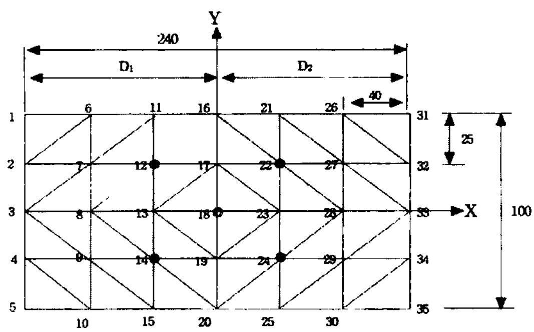
in order to minimize the following least square estimation.

$$J=\sum_{i=1}^{ns}(\Omega_j^*-\Omega_{ij})^2,\ j=1,\cdots,n_{ar} \quad (11)$$

3. Numerical Results

Aluminium plate (240 mm×100 mm×6 mm) with free boundary condition shown in Fig. 1 is implemented as fixture for numerical analysis. Fixture is divided into 48 triangular elements for finite element analysis. Density ρ, Young's modulus E, and Possion's ratio ν of the fixture are 2770(kg/m^3), 70(GPa) and 0.3 respectively. Node point 18 (Z direction) is considered as excitation point and node point 12, 14, 22, and 24 (Z direction) as 4 specimen attachment points. Structural dynamic modification is performed to coincide simultaneously with anti-resonance frequencies at 4 specimen attachment points. In order to make the anti-resonance frequencies at each specimen points of the original fixture are different, thickness is added 4 mm to element ⑲~㉒ and added 9 mm to element ㉗~㉚. The anti-resonance frequencies at node point 12 and 14 and at node point 22 and 24 are identical since the fixture is symmetric with respect to x-axis.

FRF (frequency response function) of original fixture at specimen attachment points is shown in Fig. 2. The resonance and anti-resonance frequencies are shown in Table 1. There are four resonance frequencies and two anti-resonance frequencies within the interest frequency range (5 Hz~2 kHz). But the 2nd and 4th resonance frequencies are not appeared in FRF because of nodal points.



● : Specimen attachment points
◎ : Excitation point

Fig. 1 Original plate fixture model

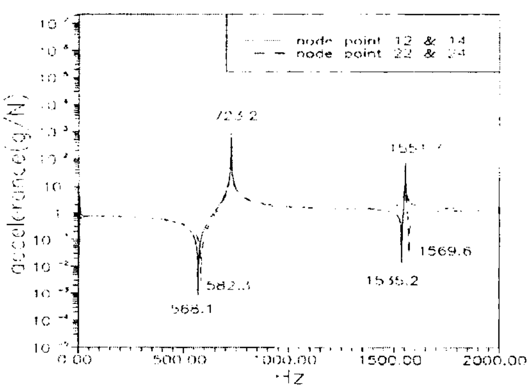


Fig. 2 FRF of original plate fixture model

3.1 Results of vibration test control of fixture before structural modification

Average control is implemented so that the PSD (power spectral density) of acceleration at the specimen attachment points become as flat as possible with 0.01 g^2/Hz. Figure 3 shows the

Table 1 Resonance and anti-resonance frequencies of original plate fixture model

	Resonance Frequency	Antiresonance Frequency	
		node 12 and 14	node 22 and 24
①	723.2 Hz	568.1 Hz	582.3 Hz
②	1118.9 Hz	1535.2 Hz	1569.6 Hz
③	1551.7 Hz	–	–
④	1874.8 Hz	–	–

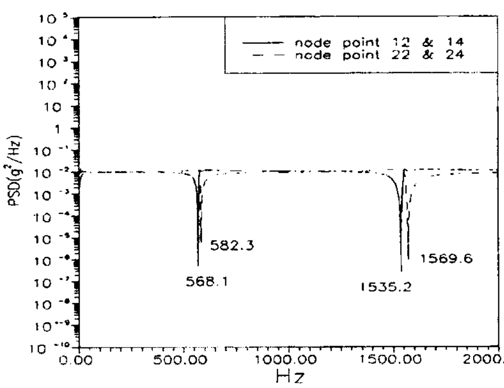


Fig. 3 Controlled spectra at specimen attachment points before modification

controlled spectra at specimen attachment points.

The results show good control at resonance frequencies, 723.2 Hz, 1551.7 Hz, which means over-tests at specimen attachment points can be removed by average control. The results, however, lead to the under-tests at 568.1 Hz, 582.3 Hz, 1535.2 Hz, and 1569.6 Hz, resulting from the inconsistency of anti-resonance frequencies of specimen attachment points.

3.2 Results of vibration test control of fixture after structural modification

Structural dynamic modification is performed using sensitivity analysis of anti-resonance frequencies to coincide with anti-resonance frequencies at specimen attachment points. Vibration test control about the modified fixture is also performed through computer simulation.

3.2.1 Results after length modification of fixture

The length of fixture, D_1 are D_2 used as the design variables in order to remove the under-tests only at the 1st anti-resonance frequency. The resonance and anti-resonance frequencies after structural modification are shown in Table 2. The 1st anti-resonance frequency is coincided at 575.0 Hz, but the 2nd anti-resonance frequencies are shifted to 1507.5 Hz and 1590.1 Hz, respectively. FRF at specimen attachment points after structural modification is shown as Fig. 4, and the modified length obtained from the sensitivity analysis is shown in Table 3.

The result of vibration test control is shown in Fig. 5. It shows that the under-tests at the 1st anti-resonance frequency is removed. However, the under-tests at the 2nd anti-resonance frequency is still uncontrolled.

Table 2 Resonance and anti-resonance frequencies after length modification

	Resonance Frequency	Antiresonance Frequency	
		node 12 and 14	node 22 and 24
①	722.8 Hz	575.0 Hz	575.0 Hz
②	1119.9 Hz	1507.5 Hz	1590.1 Hz
③	1547.4 Hz	–	–
④	1869.8 Hz	–	–

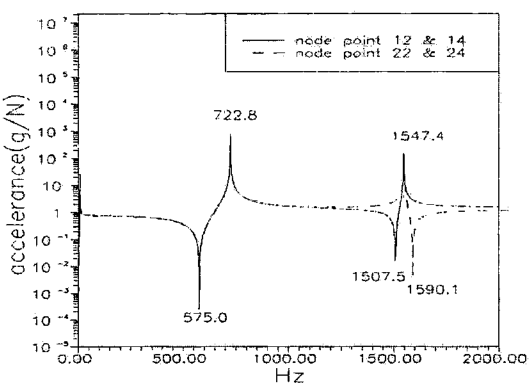


Fig. 4 FRF after length modification

3.2.2 Results after thickness modification of fixture

In order to control the fixture perfectly within the interest frequency range, structural dynamic modification is performed so that both the 1st and 2nd anti-resonance frequencies, as referred to Table 1 and Fig. 5, have to be coincided. The thickness of 12 regions shown in Table 4 are adopted as design variables.

Table 3 Design variables after length modification

Design Variables	Values
D_1	117.69
D_2	122.57

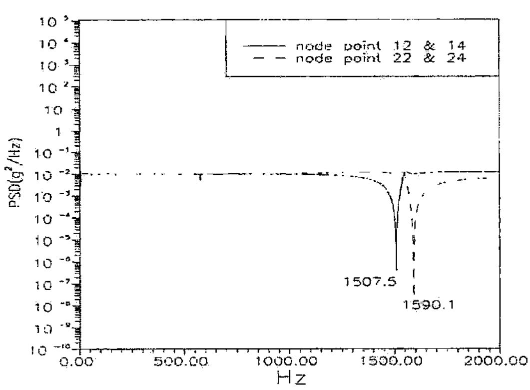


Fig. 5 Controlled spectra at specimen attachment points after length modification

Table 4 Design variables and element numbers

Group	1	2	3	4	5	6
Element Number	①, ②, ⑦, ⑧	③, ④, ⑤, ⑥	⑨, ⑩, ⑮, ⑯	⑪, ⑫, ⑬, ⑭	⑰, ⑱, ㉓, ㉔	⑲, ⑳, ㉑, ㉒
Group	7	8	9	10	11	12
Element Number	㉕, ㉖, ㉛, ㉜	㉗, ㉘, ㉙, ㉚	㉝, ㉞, ㊴, ㊵	㉟, ㊱, ㊲, ㊳	㊶, ㊷, ㊼, ㊽	㊸, ㊹, ㊺, ㊺

Table 5 Resonance and anti-resonance frequencies after thickness modification

	Resonance Frequency	Antiresonance Frequency	
		node 12 and 14	node 22 and 24
①	723.1 Hz	574.4 Hz	574.4 Hz
②	1112.0 Hz	1552.3 Hz	1552.3 Hz
③	1552.3 Hz	–	–
④	1877.3 Hz	–	–

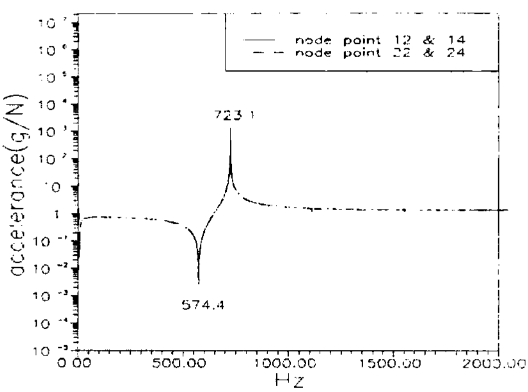


Fig. 6 FRF after thickness modification

Table 6 Plate Thickness after Modification [Unit : mm]

Group	1	2	3	4	5	6
Thickness	5.70	5.76	5.98	5.73	6.38	6.99
Group	7	8	9	10	11	12
Thickness	4.89	5.22	6.40	5.92	6.98	6.73

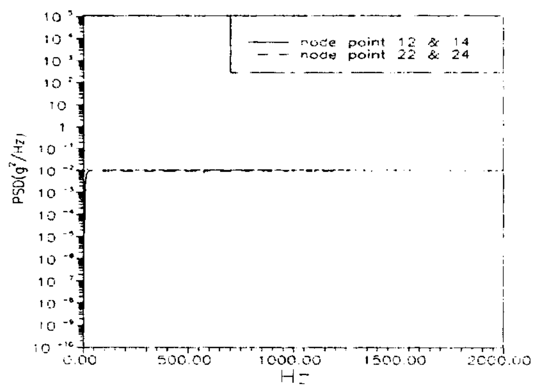


Fig. 7 Controlled spectra at specimen attachment points after thickness modification

Anti-resonance frequencies of the modified fixture are shown in Table 5. After structural modification, both the 1st and 2nd anti-resonance frequencies are coincided with each other at 574.4 Hz and 1552.3 Hz. FRF of the modified fixture at specimen attachment points is shown in Fig. 6. The 3rd resonance frequency and the 2nd anti-resonance frequency become identical in the modified fixture as shown in Table 5. So the peak does not appear in FRF due to the cancellation of resonance and anti-resonance frequency. The thickness of the modified fixture is shown in Table 6. Figure 7 shows the vibration test control result. This result makes it possible to control the specimen attachment points in accordance with the specified reference spectrum. All the response spectra at the specimen attachment ponts are controlled perfectly to be same and flat throughout the interest frequency range. The under-test in vibration test control is removed after structural modification of the fixture.

4. Conclusions

The conclusions of this study are as follows :

(1) The sensitivity analysis of anti-resonance frequency in multi-input system is proposed and applied to the vibration test fixture.

(2) Computer simulation results show that the under-tests at anti-resonance frequencies can be removed by the structural modification of the

fixture using the sensitivity analysis of anti-resonance frequency.

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